Semester-II (FYUGP)

Paper: Electricity and Magnetism

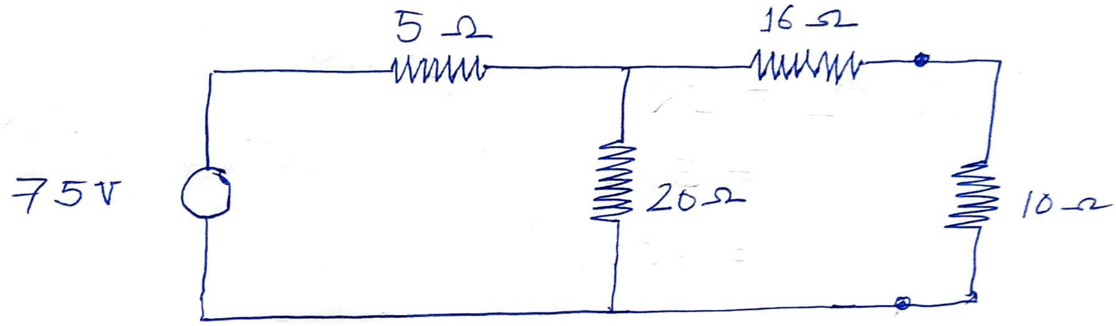
Paper Code: PHY0200104

Unit-V (Network Theorems)

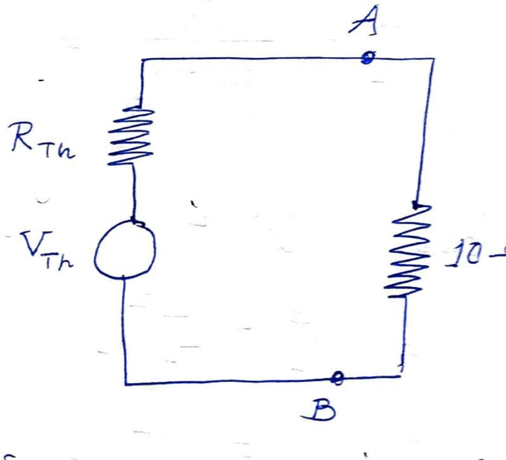
BY

Dr. S. Zamal

Ex: Transform the following circuit into Thevenin's equivalent circuit, and hence find the value of (a) Thevenin's equivalent ~~em~~ impedance, (b) Thevenin's equivalent voltage source, and (c) load current and power in the 10 Ω resistor. [TDC, Sem-V, GU-2016]

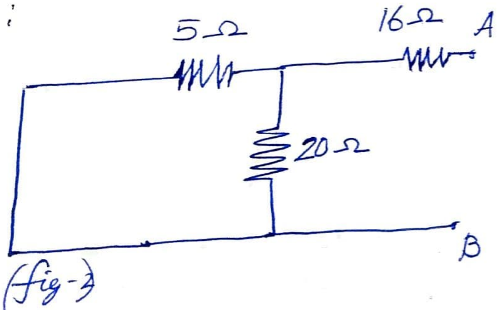


Solution: Thevenin's equivalent circuit is shown in fig-1.



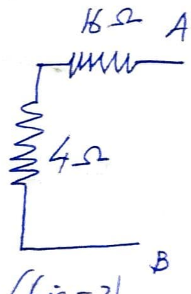
(fig-1)

(a) Thevenin's equivalent impedance R_{Th} is calculated as follows:



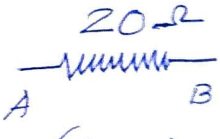
(fig-2)

⇒



(fig-2)

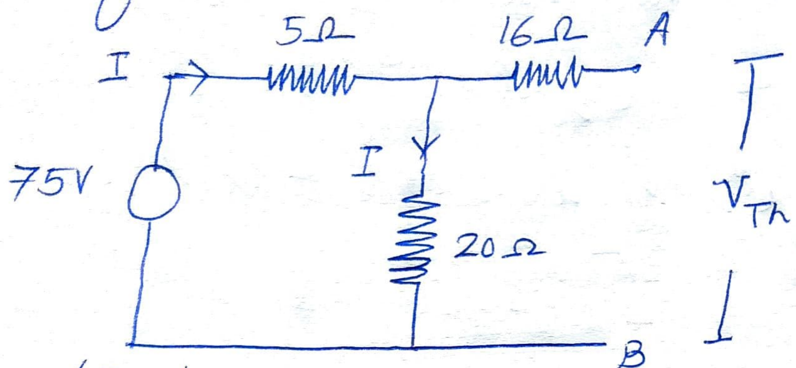
⇒



(fig-3)

Thus $R_{Th} = 20\ \Omega$

(b) Thevenin's equivalent voltage V_{Th} is calculated as follows :



(fig-4)

Here $I = \frac{V}{R}$

$= \frac{75}{5+20}$

$= 3\ A.$

$\therefore V_{Th} = IR$

$= 3 \times 20$

$= 60\ V.$

(c) Current through the load resistor (from f

is $I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{60}{20 + 10} = 2\ A.$

$\therefore$ Power dissipated at R_L is

$P_L = I_L^2 R_L$

$= 2^2 \times 10$

$= 40\ W.$

Ex: An AC source of emf 40 V delivers a maximum power of 10 W to an external load resistance. What is the internal resistance of the source?

How much current will be drawn from the source if the output terminals are short-circuited?

Solution: The networks contain only resistances. So

$$P_{max} = \frac{E_{rms.}^2}{4R_0}$$

For max power

$$P = \frac{E_{rms}^2 R_L}{(R_0+R_L)^2 + (X_0+X_L)^2}$$

Here $X_0 = X_L = 0$

$$\therefore P = \frac{E_{rms}^2 R_L}{(R_0+R_L)^2}$$

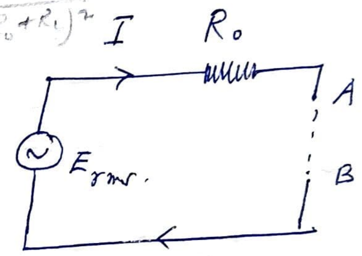
But $R_L = R_0$

$$\therefore P_{max} = \frac{E_{rms}^2}{4R_0^2}$$

$$\Rightarrow R_0 = \frac{E_{rms.}^2}{4 P_{max.}}$$

$$= \frac{40 \times 40}{4 \times 10}$$

$$= 40\,\Omega.$$



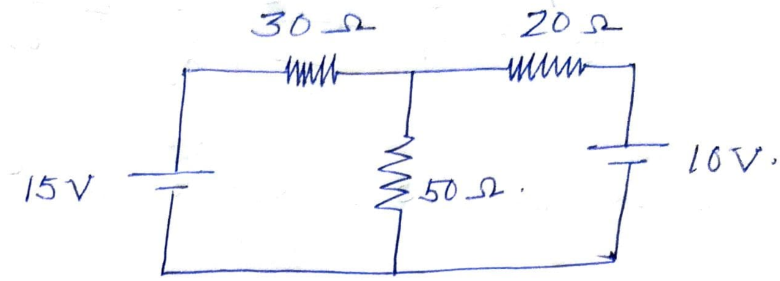
The short circuit current will be

$$I = \frac{E_{rms.}}{R_0}$$

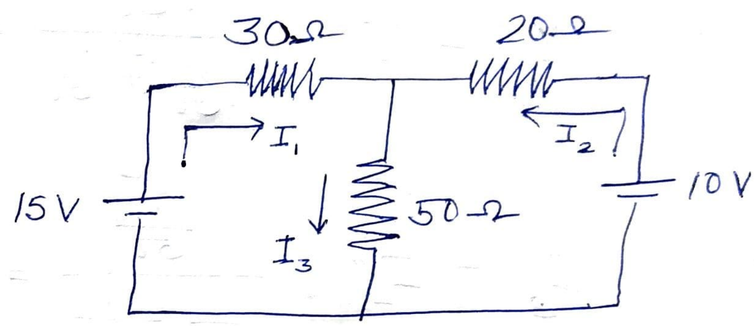
$$= \frac{40}{40}$$

$$= 1\text{ A}.$$

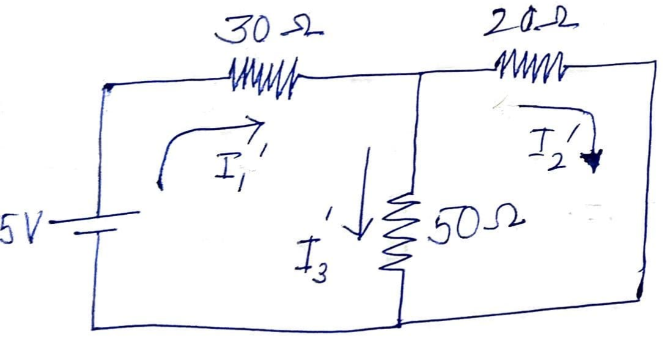
Ex.: Apply superposition principle to determine the currents flowing through each branch of the network. Verify the result using Kirchhoff's laws.



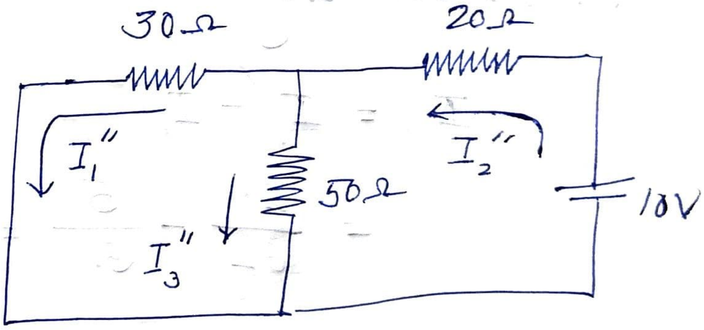
Solution: Let I_1, I_2 and I_3 be the currents through the 30 Ω, 20 Ω and the 50 Ω resistors respectively (fig-1).



(fig-1)



(fig-2)



(fig-3)

In fig-2, the 10 V source is removed. The currents are

$$I_1' = \frac{15}{30 + \frac{\cancel{20} \times \cancel{50}}{\cancel{20} + \cancel{50}}} = \frac{15}{30 + \frac{100}{7}} = \frac{\cancel{105}^{\,21}}{\cancel{310}_{\,62}} = \frac{21}{62} \text{ A.}$$

Using current division rule we get

$$I_2' = I_1' \frac{\frac{1}{20}}{\frac{1}{20} + \frac{1}{50}} = \frac{\cancel{21}^{\,3}}{62} \times \frac{1/20}{7/100} = \frac{\cancel{21}^{\,3}}{62} \times \frac{5}{7} \text{ A} = \frac{15}{62} \text{ A}$$

$$I_3' = I_1' \times \frac{\frac{1}{50}}{\frac{1}{50}+\frac{1}{20}} = \frac{\cancel{21}^{3}}{62} \times \frac{1/\cancel{50}}{\cancel{7}/\cancel{100}_{2}} = \frac{3}{31}\ A.$$

Now the 15 V source is removed (fig-3). We have

$$I_2'' = \frac{10}{20+\frac{30\times 5\cancel{0}}{3\cancel{0}+5\cancel{0}}} = \frac{10}{20+\frac{\cancel{150}\,75}{\cancel{8}\,4}} = \frac{\cancel{10}^{2}\times 4}{\cancel{155}_{31}} = \frac{8}{31}\ A.$$

Current division rule gives

$$I_1'' = I_2'' \times \frac{1/30}{1/30+1/50} = \frac{\cancel{8}}{31} \times \frac{\frac{1}{\cancel{30}}}{\frac{\cancel{8}}{\cancel{150}_{5}}} = \frac{5}{31}\ A.$$

$$I_3'' = I_2'' \times \frac{\frac{1}{50}}{\frac{1}{50}+\frac{1}{30}} = \frac{\cancel{8}}{31} \times \frac{\frac{1}{\cancel{50}}}{\frac{\cancel{8}}{\cancel{150}_{3}}} = \frac{3}{31}\ A.$$

∴ Current through the 30 Ω resistor is

$$I_1 = I_1' - I_1''$$

$$= \frac{21}{62} - \frac{5}{31} = \frac{11}{62}\ A.$$

$$I_2 = I_2' - I_2''$$

$$= \frac{15}{62} - \frac{8}{31} = \frac{15-16}{62} = -\frac{1}{62}\ A.$$

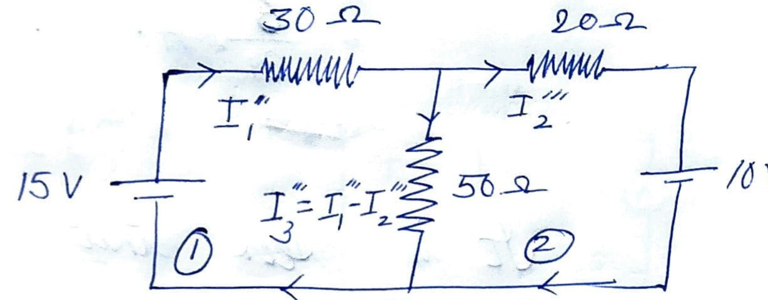
i.e. $I_2 = \frac{1}{62}$ A in the anticlockwise direction.

$$I_3 = I_3' + I_3''$$

$$= \frac{3}{31} + \frac{3}{31} = \frac{6}{31}\ A$$

Verification by Kirchhoff's laws.

Current distribution in the three branches using KCL is shown in fig-4.



(fig-4)

Using KVL to mesh ① we get

$$-\overset{6}{\cancel{30}}\, I_1'' - \overset{10}{\cancel{50}}\,(I_1''-I_2'') + \overset{3}{\cancel{15}} = 0.$$

$$\Rightarrow\ 6I_1'' + 10\,(I_1''-I_2'') = 3$$

$$\Rightarrow\ 16\,I_1'' - 10\,I_2'' = 3 \longrightarrow (1)$$

Using KVL to mesh ② we get.

$$-\overset{2}{\cancel{20}}\, I_2''' + \overset{5}{\cancel{50}}\,(I_1''-I_2''') - \overset{1}{\cancel{10}} = 0.$$

$$\Rightarrow\ 5\,I_1''' - 7\,I_2''' = 1 \longrightarrow (2)$$

Using Cramer's rule we get

$$I_1''' = \frac{\begin{vmatrix} 3 & -10 \\ 1 & -7 \end{vmatrix}}{\begin{vmatrix} 16 & -10 \\ 5 & -7 \end{vmatrix}} = \frac{-21+10}{-\cancel{11}2+50} = \frac{-11}{-62} = \frac{11}{62}\ A.$$

$$I_2'' = \frac{\begin{vmatrix} 16 & 3 \\ 5 & 1 \end{vmatrix}}{\begin{vmatrix} 16 & -10 \\ 5 & -7 \end{vmatrix}} = \frac{16-15}{-62} = -\frac{1}{62}\ A.$$

i.e $I_2''' = \frac{1}{62}$ A in anticlockwise direction

And $I_3''' = I_1''' - I_2'''$

$= \frac{11}{62} - \left(-\frac{1}{62}\right)$

$= \frac{\cancel{12}\,6}{\cancel{62}\,31}$

$= \frac{6}{31}$ A.

It is seen that $I_1''' = I_1$

$I_2''' = I_2$

and $I_3''' = I_3$

verified.